# Imaging of nonlinear materials via Monotonicity Principle

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In this paper, we focus on Electromagnetic Tomography, where the aim is to reconstruct the interior of a body exciting the sample under test with a certain form of electromagnetic energy and measuring a proper electromagnetic quantity on the boundary. Specifically, we consider low-frequency excitation. In this area, the Monotonicity Principle (MP) has a relevant role. MP is a quite general property shared by a large variety of different problems. Basically, MP states a monotone relationship between the values of an unknown material property in the interior of the body and the measurements made on the boundary. In 2002 Tamburrino and Rubinacci recognized a MP in Electrical Resistance Tomography as the basis of a new family of non-iterative imaging methods for the inverse obstacle problem, capable to conjugate excellent performances and real-time operations. Then, in the following years, a MP was discovered for Eddy-Current Tomography (small and large skin depth regimes in frequency domain operations, time constants and transfer function in time-domain operations)and for Wave Propagation problems. In ERT, the excitation is given by the voltages imposed on the boundary of the conductive domain of interest, while the measured quantity is the resulting electric current on the boundary, in DC operations. Let A be the anomalous region occupied by a material more conductive than the background, and let T be an arbitrary region, called test anomaly, made by the same material. The MP gives that if the test anomaly T is included in the region A then the power absorbed in the configuration with the test anomaly (Pt) is less than the power absorbed when the actual anomaly is considered (Pa), for every possible voltages at the boundary. Tamburrino and Rubinacci derives their method from the negation of the previous statement. In other words, the basic observation is that if it is possible to find a proper excitation such that the power absorbed in presence of T is greater than the power absorbed in presence of A than surely T is not included in A. This is the so-called elementary monotonicity test. A possible estimate of the anomaly is, hence, given by the union of all test anomalies for which Pt is less than Pa, for every excitation. Recently, the MP has been generalized to nonlinear materials, under very mild assumptions. Also in this case, the MP takes some form as for the linear case, but the measured quantity (a proper average of the absorbed power) has a nonlinear dependence with respect to the voltages at the boundary. This represents a very significant challenge in setting up inversion methods based on the MP. Indeed, while for linear materials the elementary monotonicity test can be taken back to an eigenvalue problem, no such general mathematical tools are available for nonlinear operators. In our work, we propose a new non-iterative inversion method for reconstructing the shape of nonlinear anomalies embedded in a given linear background, able to overcome the difficulties above mentioned. Specifically, we evaluate, before the measurements, a set of optimal excitations to perform the elementary monotonicity tests. In this way, each test requires a low number of measurements, i.e. the method Is feasible for real-time applications, preserving one the most significant feature of MP methods for linear materials. References A. Tamburrino and G. Rubinacci, A new non-iterative inversion method for electrical resistance tomog- raphy, Inverse Problems, 18 (2002), pp. 1809–1829. A. Tamburrino and G. Rubinacci, Fast methods for quantitative eddy-current tomogra- phy of conductive materials, IEEE Transactions on Magnetics, 42 (2006), pp. 2017–2028. Z. Su, L. Udpa and A. Tamburrino, Monotonicity of the transfer function for Eddy Current Tomography, Submitted, 2023. A. Tamburrino, L. Barbato, D. Colton, P. Monk, Imaging of Dielectric Objects Via Monotonicity of the Transmission Eigenvalues, abstracts book of the 12th Int. Conf. on Mathematical and Numerical Aspects of Wave Propagation, pp. 99-100, Karlsruhe (Germany), July 20-24, 2015. A. Albicker and R. Griesmaier, Monotonicity in inverse obstacle scattering on un- bounded domains, Inverse Problems, (2020). A. Corbo Esposito, L. Faella, G. Piscitelli, R. Prakash and A. Tamburrino, Monotonicity principle in tomography of nonlinear conducting materials, Inverse Problems, Vol 37, No 4, pp 045012, Apr. 2021. V. Mottola, A. Corbo Esposito, G. Piscitelli, A. Tamburrino, Imaging of nonlinear materials via the Monotonicity Principle, submitted, 2023.